



Determination of an appropriate model for optimum use of N fertilizer in furrow irrigation

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ABSTRACT

Estimation of optimum fertilizer application is of interest because of production cost, environmental concerns, and performance increase. Optimum fertilizer rates can be determined by fitting statistical models to yield data collected from N fertilizer experiments. Quadratic polynomial, square root, Mitscherlich, hyperbolic trigonometry, quadratic polynomial threshold, and threshold linear models which describe the production function of sugar beet to different rates of N fertilization (0, 60, 120, 180 and 240 kg/ha) were evaluated with supplemental furrow irrigation in three replications in each of two years (2003 to 2004) of experiment. Economic, optimum N rates were estimated based on both fertilizer and crop costs in two years. The fertilizer rate was variable based on fertilizer cost to crop cost and also the applied model. Results showed that the quadratic polynomial model is the most appropriate model for describing the production function and optimal N application in sugar beet production. Using this model, the economic, optimum N rates in 2003 and 2004 were 235.8 and 248.9 kg/ha, respectively.

Keywords: nitrogen fertilizer, quadratic polynomial model, economic consumption, sugar beet

INTRODUCTION

Since N is one of the most important elements which is taken up more than other elements, estimation of the optimal rate of N fertilizer in sugar beet production is of particular importance (Weeden 2000). The type of N application and its rate has a special significance on the quantity and quality of sugar beet (Hills et al. 1978). High level of N absorption from the soil will increase the root impurities and may decrease the sugar extraction (Cattanach et al. 1993). Nitrogen is a mobile and easily washable element (Cattanach et al. 1993). Therefore, excess or inappropriate application of N, not only may decrease the fertilizer efficiency but also is the most important source of groundwater pollution (Hills et al 1978). Data from fertilizer studies can be fitted to several statistical models to determine optimum fertilizer rates

(Cerrato and Blackmer 1990; Belanger et al. 2000; Sayili and Akca 2004; Mortensen and Beattie 2005). The selection of the most appropriate model for estimating the optimal rate of N was evaluated in different crops such as maize (Cerrato and Blackmer 1990; Bullock and Bullock 1994; Sumelius et al. 2002; Franke et al. 2004), potato (Belanger et al. 2000), lettuce (Pour Marvi 2008), sugar beet (Adams et al. 1983; Sayili and Akca 2004; Lim et al. 2010), sorghum, soybean (Wortmann et al. 2007), spring and winter barley, and winter wheat (Mortensen and Beattie 2005). The selection of an appropriate model is so important due to its influence on estimation of the optimal rate of fertilizer production function (Cerrato and Blackmer 1990; Belanger et al. 2000; Sayili and Akca 2004; Mortensen and Beattie 2005; Pour Marvi 2008). Different models such as quadratic polynomial, square root, Mitscherlich, hyperbolic trigonometry, quadratic equation threshold, and

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Table 1. Average of analysis on some soil parameters of the experimental field at the depth of 0-30 cm in two years of the experiment

Year	Electrical conductivity	Acidity	Neutral materials (%)	Organic carbon (%)	Total N (%)	Absorbable Phosphorus (mg/kg)	Absorbable potassium (mg/kg)	Silt (%)	Loam (%)	Sand (%)	Profile
2004	0.43	8.6	10.35	0.47	0.47	13.2	400	24.4	31.9	43.7	L
2005	0.41	8.8	4.95	0.3	0.3	11.6	290	14.7	23.9	61.4	SL

threshold linear were used for estimating optimal fertilizer application (Cerrato and Blackmer 1990; elanger et al. 2000; Sayili and Akca 2004; Mortensen and Beattie 2005; Pour Marvi 2008; Franke et al. 2004; Sumelius et al. 2002; Adams et al. 1983). In study by Adams et al. (1983) the optimal rate of fertilizer in sugar beet production was estimated using a quadratic polynomial model. Applied N was between zero to 448 kg/ha. The applied N for the maximum production, based on root yield, in each of three years were 224, 238 and 249 kg/ha and for economic optimum production, the rates were 214, 205 and 240 kg/ha, respectively. Sayili and Akca (2004) fitted nine linear models such as quadratic polynomial, square root, exponential, semi-logarithmic, third degree polynomial, Cob Douglas and reciprocal based on information collected from 75 sugar beet fields in two years (2002 to 2003). Based on correlation coefficient and standard error, quadratic polynomial was found appropriate and the economic, optimum fertilizer rate was 321.07 kg/ha. Lim et al. (2010) fitted the results of sugar beet experiment in pot cultivation containing saline sodic soil to four production function models including quadratic polynomial, exponential, square root and linear with threshold. The linear with threshold model had the best fit with data and the optimum rate of N was 138 kg/ha. In a study by Talleghani et al. (1998) on water and N use efficiency in sugar beet, the required N fertilizer for achieving the maximum sugar yield in Karaj region was 240 kg/ha. Hills et al. (1978) reviewed the data collected from 21 sugar beet fields and concluded that with increase of 18 kg N fertilizer per hectare, one ton root per hectare was produced. Bilbao et al. (2004) conducted thirty three experiments on sugar beet in areas with limited drainage, water shortage, and limited irrigation with eight N fertilizer rates including 0, 40, 80, 120, 160, 200, 240 and 300 kg/ha and the estimated approximate total rate of N required for optimal sugar beet production was 268 kg/ha.

Sugar beet was planted in 3190 hectare in Hamedan province in 2010-2011 (Anon 2011) which had a decrease of 55% compared with 2005-2006

with a planted area of 7111 ha (Anon 2006) but had 9.2 times increase compared with 2009-2010 with a planted area of 1106 ha (Anon 2010a). Due to the potential facilities provided in the province, the increase in planting area in the following years is expected. However, as pointed before, N is one of the most important elements in sugar beet production which also was applied more than other elements.

This study was conducted to evaluate the appropriate model for sugar beet production function and to estimate the optimum rate of N application in Hamedan province in order to increase the yield of sugar beet and to decrease production costs and also counteracting the detrimental effects on environment.

MATERIALS AND METHODS

To estimate the economic and optimum fertilizer rate, the data collected from Rezvani et al. (2009) study on fertilizer rate and sugar beet root yield in furrow irrigation were used. This study was conducted in Hamedan Agricultural and Natural Resources Research Center in split plot design with three replications and 15 treatments in 2004 to 2005. Different irrigation systems including sprinkler irrigation (classic), furrow irrigation (Hydroflume) and drip (tape) irrigation using tape (508-20-500) were considered as main plots and five N fertilizer (Pure N) rates of 0, 60, 120, 180 and 240 kg/ha as subplots. In this paper, only the result of root yield from furrow irrigation which is the most common irrigation method in Hamedan area was used. For this experiment, a planting pattern of 50×40 (plants spaced 40 cm apart on the rows and 50 cm between rows) was used. Each subplot had an area of 108 m² and planting was in a row pattern (furrow and hill, two rows on each hill, four hills in each plot) and 30 m long. In the first and second years, the soil profile was loam and sandy loam, respectively (Table 1). Monogerm cultivar, Dorotea was used and phosphorous and potassium fertilizers were applied based on the soil test results. N fertilizer as urea was top-dressed in each plot. Seeds were planted using pneumatic planter. At harvest time, root

sampling from each plot was done using two random boxes (each box contained two lines with a 5 m length).

Sumelius et al. (2002) and Mostensen and Beattie (2003) used function 1 for optimising the profit rate:

$$\Pi = P_i f(x_n, x_1, \dots, x_z, s, r) - wx_1 \quad (1)$$

where $y = f(x_n, x_1, \dots, x_z, s, r)$ is the output, x_n is applied N, and $(x_n, x_1, \dots, x_z, s, r)$ is inputs except N rate, s = soil, r = rainfall and w = the price of the input i .

Assuming that all production inputs are deterministic, with any changes in N rate, the maximum benefits from x_n rate of applied N will be gained from the first derivative function in description of the second equation:

Using this simple model, the optimum N fertilizer rate can be achieved. Approved rate per kg urea fertilizer for the years 2004 and 2005 were 420 and 450 Rials, respectively (Anon 2012b). In 2012, the approved rate per kg urea was 1350 Rials (Act No. 115050/T/46416 dated 29.08.2011, approved by the Council of Ministers) and the rate of urea fertilizer in the free markets by late 2012 based on questions asked from farmers was 3000 Rials. For the guaranteed purchase of sugar beet with a sugar content of 16% in 2004 and 2005 the price was 355 and 390 Rial per kg, respectively and in 2012 reached to 900 Rials per kg (Anon 2011). Models used in this study, in which Y is a corresponding function with different rates of N, are from equations number 3 to 8:

$$\begin{aligned} \frac{\partial \pi}{\partial x_n} &= \frac{\partial P f(x_n, x_1, \dots, x_z, s, r)}{\partial x_1} - \frac{\partial w_1 x_1}{\partial x_1} = 0 \\ \frac{\partial \pi}{\partial x_n} &= P \frac{\partial f(x_n)}{\partial x_n} - w_1 = 0 \\ \frac{\partial f(x_n)}{\partial x_n} &= \frac{w_1}{P} \end{aligned} \quad (2)$$

Quadratic polynomial

$$Y = \beta_0 + \beta_1 N + \beta_2 N^2 \quad (3)$$

where β_0 is the width of the source, β_1 is the linear coefficient, and β_2 is the quadratic coefficient (Cerrato and Blackmer 1990; Belanger et al. 2000; Sayili and Akca 2004; Mortensen and Beattie 2005; Pour Marvi 2008; Adams et al. 1983).

Square root model

$$Y = \beta_0 + \beta_1 N^{(0.8)} + \beta_2 N \quad (4)$$

where β_0 is the width of the source, β_1 is the linear coefficient, and β_2 is the quadratic root coefficient (Cerrato and Blackmer 1990; Belanger et al. 2000; Sumelius et al. 2002; Sayili and Akca 2004; Adams et al. 1983).

Hyperbolic trigonometry

$$Y = \beta_0 + \frac{\beta_1}{1 + \beta_2 N} \quad (5)$$

where β_0 is the highest limit of performance while N tends to infinity ($N \rightarrow \infty$), $\beta_0 + \beta_1$ is the point of coincidence between curve and y-axis (mean yield) while $N=0$, and β_2 is the parameter of the figure (Franke et al. 2004).

Mitscherlich model

$$Y = \beta_0 (1 - e^{-\beta(N+\beta_1)}) \quad (6)$$

where the maximum performance available (β_0) equals to 99.9% while $N \rightarrow \infty$ situation, β_2 and β_1 are the fixed coefficients obtained from the fitting of the model to the data. β_2 coefficient represents the effect of Mitscherlich (Cerrato and Blackmer 1990; Belanger et al. 2000; Sumelius et al. 2002).

Quadratic equation threshold

$$Y = \beta_0 + \beta_1 N + \beta_2 N^2 \quad \text{for } N < C \quad (7)$$

$$Y = P \quad \text{for } N \geq C$$

where β_0 is the width of the source, β_1 is the linear coefficient, and β_2 is the quadratic coefficient, C is the critical rate of fertilizer which occurs in the point of coincidence between quadratic production function and linear constant of the curve and P is the threshold yield (Cerrato and Blackmer 1990; Sayili and Akca 2004; Mortensen and Beattie 2005).

Linear model with threshold

$$Y = \beta_0 + \beta_1 N \quad \text{for } N < C \quad (8)$$

$$Y = P \quad \text{for } N \geq C$$

where β_0 is the width of the source, β_1 is the linear coefficient, C is the critical rate of fertilizer which

Table 2. Root yield of sugar beet at different N rates in two years

Year	Treatment	Root yield (t/ha)		
		Rep 1	Rep 2	Rep 3
2004	0 N	40.56	33.78	43.22
	60 N (t/ha)	39.11	47.11	49.56
	120 N (t/ha)	43.89	47.44	53.22
	180 N (t/ha)	54.44	63.33	59.44
	240 N (t/ha)	52.67	54.44	53.89
2005	0 N	50.00	51.44	47.44
	60 N (t/ha)	60.22	62.44	78.11
	120 N (t/ha)	63.00	70.44	75.67
	180 N (t/ha)	65.00	67.11	77.89
	240 N (t/ha)	75.00	77.33	84.89

occurs in the point of coincidence between linear production function and linear constant of the curve and P is the threshold yield (Cerrato and Blackmer 1990; Pour Marvi 2008).

For assessing the validity of the regression models, regression assumptions including normal distribution of residuals (Shapiro-Wilk), autocorrelation test (Durbin Watson) and heterogeneity of residuals variance were run by SigmaPlot v. 12 software (Anon 2010c).

RESULTS

Combined analysis of variance showed that there was a significant difference between root yields in two years of the experiment (Rezvani et al. 2009). Therefore, the results of each year were analysed separately (Table 2). Coefficients of the fitted models on results of fertilizer rates and root yield in 2004 and 2005 are given in Table 3. The coefficient of determination was found to be higher for quadratic threshold followed by first degree threshold and quadratic polynomial, respectively, in 2004. In this year, the lowest rate of the above coefficient belonged to square root model, whereas, this model showed the highest coefficient of determination and threshold linear model with the lowest coefficient of determination. In model selection, coefficient of determination is a weak criterion for estimating the parameters (Cerrato and Blackmer 1990; Belanger et al. 2000). However, all models had different coefficients of determination but all fitted models were statistically significant at less than or equal to 0.01 (Table 3).

Regression models to be validated should not have any systematic bias, hence the regression residuals should have a normal distribution (Cerrato and Blackmer 1990). The normality test for the residuals was done through Shapiro-Wilk method

Table 3. Fitted coefficients on sugar beet yield models with different rates of N fertilizer applied in 2004 and 2005

Year	Model	Coefficients			Coefficient of determination (R ²)	Statistic F	Standard error (SE)	Root mean square error	Spearman's rank correlation (ρ)	Durbin Watson	Shapiro-Wilk test	
		B ₀	B ₁	B ₂							P	W
2004	Quadratic equation	38.3757	0.1477	0.0003	0.6669	12.0109**	4.99	4.46	0.3809*	2.2463	0.9757**	0.9810
	Root	38.8787	0.0256	0.7485	0.6362	10.4905**	5.21	4.66	0.9234*	2.0682	0.8663**	0.9686
	Mitscherlich	61.1605	0.0063	159.0926	0.6531	11.2952**	4.89	4.55	0.6295*	2.1573	0.8347**	0.9685
	Hyperbolic trigonometry	74.3704	0.0041	-35.5350	0.6494	11.1120**	4.92	4.58	0.7143*	2.1336	0.7915**	0.9658
	Quadratic model equation with threshold	39.7744	0.0003	0.0442	0.758	679.318**	4.70	4.21	0.3809*	2.3184	0.7064*	0.9608
2005	Linear model with threshold	38.539	-	0.1043	0.739	1021.437**	4.60	4.28	0.3809*	2.3737	0.9075**	0.9736
	Quadratic equation	51.8561	-0.0004	0.2016	0.6589	11.5878**	7.12	6.37	0.2243*	2.2867	0.2961**	0.9324
	Root	49.9887	2.2603	-0.0393	0.7097	14.6648**	6.51	5.82	0.3321*	2.4248	0.5160**	0.9495
	Mitscherlich	80.7638	0.0173	-30.9867	0.6927	13.5275**	6.70	5.99	0.4894*	2.3121	0.4384**	0.9442
	Hyperbolic trigonometry	75.0696	0.0152	72.0591	0.7034	14.2318**	6.58	5.89	0.4412*	2.3486	0.4840**	0.9474
2005	Quadratic model equation with threshold	50.232	-0.0011	0.319	0.705	527.581**	7.11	6.36	0.2995*	2.1936	0.5650**	0.952
	Linear model with threshold	52.048	-	0.167	0.630	681.95**	7.35	6.57	0.3595*	2.1806	0.1802**	0.9181

significant, * p<0.05, **p<0.001

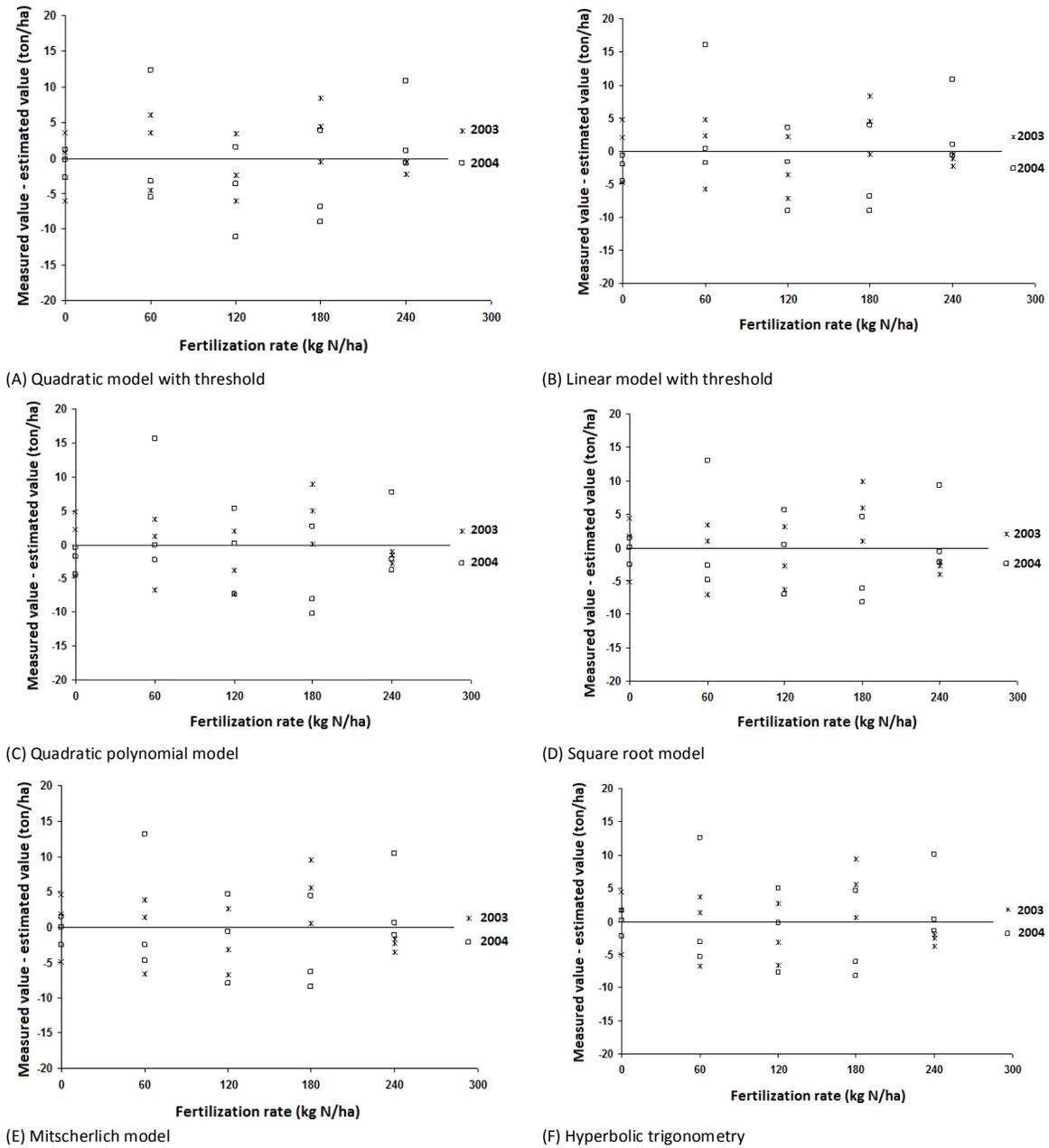


Fig. 1. The distribution of residuals in sugar beet root yield models at different rates of fertilizer

and showed that the residuals of all applied models had a normal distribution. The parameter of Durbin Watson (Table 3) in all models was about 2 which indicated the lack of autocorrelation among the studied models. The residual variance heterogeneity test which was computed through the estimation of Spearman correlation coefficient between the absolute values and observed values of the dependent variable using SigmaPlot 12 software, showed lack of dissimilarity of variance among all models.

The calculation of standard error showed that

in 2004, the lowest standard error was for threshold linear model equal to 4.60 and the maximum standard error was for square root model with a value of 5.21. In 2005, square root and linear models had the minimum (6.32) and the maximum standard error (7.12), respectively. The minimum root mean square error of the models in 2004 and 2005 belonged to quadratic threshold and square root models with 4.21 and 5.82, respectively.

The results showed that all the fitted models were statistically significant and the statistical

Table 4. The maximum and optimum rates of applied N fertilizer in sugar beet production based on the models

Year	Model	The maximum fertilizer (kg/ha)	Optimum fertilizer rate (kg/ha)	Maximum crop (t/ha)
2004	First degree with threshold	180.0	157.8	55.0
	Quadratic with threshold	180.0	§	55.0
	Quadratic	240.2	235.8	55.5
	Root	213.7	264.1	55.3
	Mitscherlich	-	+	61.1
	Hyperbolic trigonometry	-	+	74.4
2005	First degree with threshold	160.0	131.4	74.0
	Quadratic with threshold	180.0	158.2	74.0
	Quadratic	252.0	248.9	77.3
	Root	+	+	82.5
	Mitscherlich	-	+	75.0
	Hyperbolic trigonometry	-	+	80.8

Symbols § and + show negative and high rate of fertilizer, respectively.

Table 5. The optimum rate of N fertilizer based on governmental and free markets price of urea fertilizer in 2012 in models of 2004 and 2005

Year	Model	Information of the year 2004	Information of the year 2005
2004	First degree with threshold	157.8	131.4
	Quadratic with threshold	§	157.9
	Quadratic	234.7	247.9
	Root	280.7	+
	Mitscherlich	+	+
	Hyperbolic trigonometry	+	+
2005	First degree with threshold	157.8	131.4
	Quadratic with threshold	§	155.0
	Quadratic	225.1	240.8
	Root	+	+
	Mitscherlich	+	+
	Hyperbolic trigonometry	§	§

Symbols § and + show negative and high rate of fertilizer, respectively.

distribution of their residuals was too close to a normal distribution. In 2004 and 2005, quadratic polynomial and square root models had the highest proximity to normal distribution and by considering the standard error and root mean square error in 2004, linear and quadratic threshold models, and in year 2005, square root and hyperbolic trigonometry were appropriate models. In general, quadratic and square root models were appropriate for the two years experiments, respectively. Table 4 shows the economic, optimum N rates estimated through fitting different models. In 2004, only threshold linear model, quadratic and square root models with the rates of 157.8, 235.8 and 264.1 kg/ha, respectively had acceptable responses and in 2005, linear, quadratic and quadratic threshold models with values of 131.4, 158.2 and 248.9 kg/ha, respectively, showed the proper response. In 2004, the optimum rate of N fertilizer was obtained from quadratic model with negative threshold (Fig. 2B). Values obtained from Mitscherlich and hyperbolic trigonometry models were very high and had a wide difference with studied domain (Fig. 2C and D). Also, in 2005, the

values of square root, Mitscherlich and hyperbolic models were very high. In 2012, based on approved and free markets urea fertilizer price and also guaranteed purchase price, the economic, optimum applied N rate was obtained. As Table 5 shows, based on approved price of urea fertilizer in 2012, the models response was similar to 2004 and 2005 though the values differed based on variation in fertilizer N cost to crop cost. According to free markets of urea and the models used in 2005, not only the obtained values were from quadratic model with negative threshold but also the values of hyperbolic trigonometry model were negative. Large quantities were obtained through square root and Mitscherlich models. Using models of the year 2005 and free markets urea price, the behaviour of the models were similar to year 2005, except of the value obtained from hyperbolic trigonometry.

DISCUSSION

In this study, the quadratic and square root models were recognized appropriate for description of root production function to N fertilizer

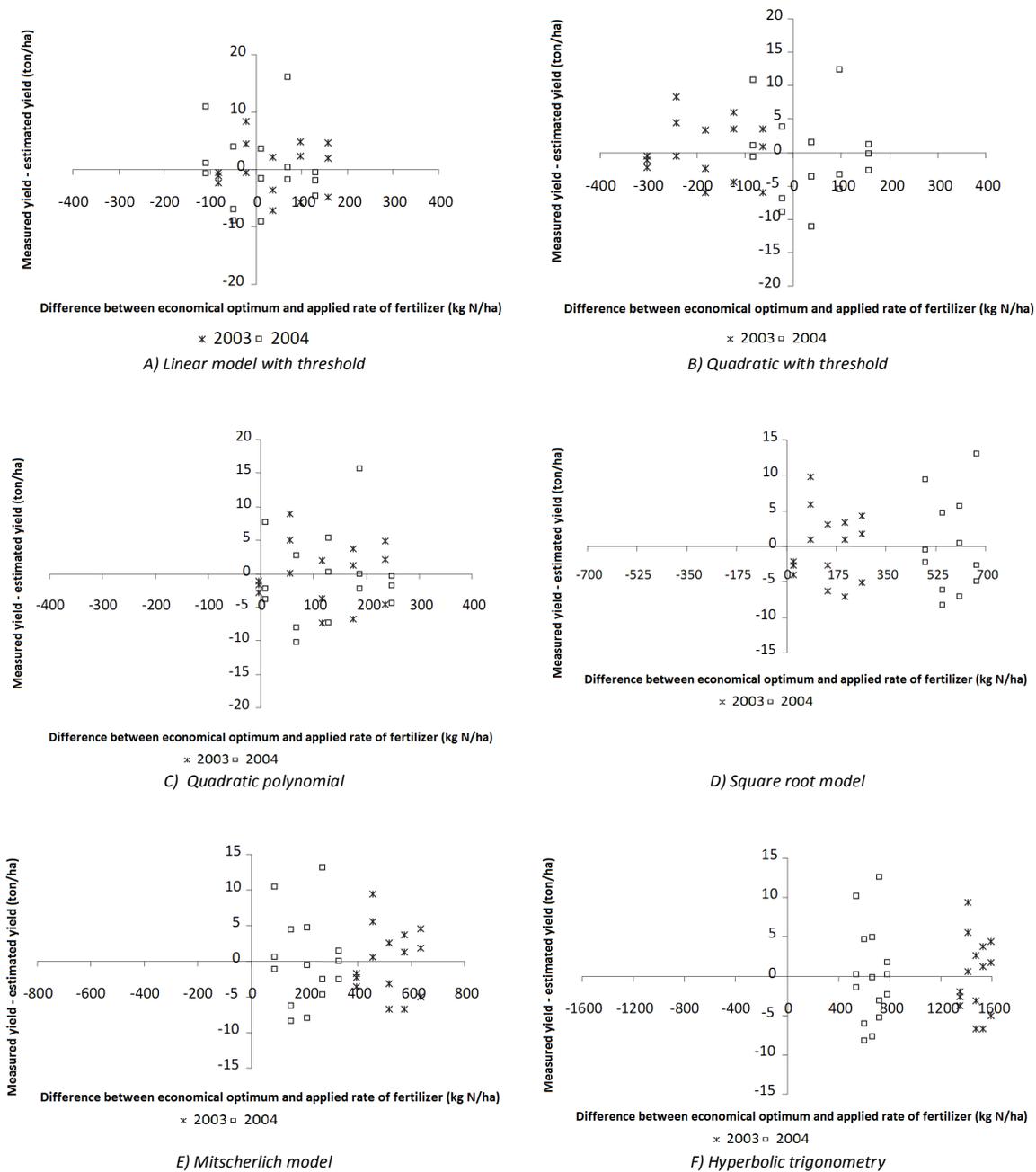


Fig. 2. Difference between economic, optimum and applied rate of N fertilizer to measured rates minus estimated rates

which corresponds to the results of Adams et al. (1983) and Sayili and Akca (2004) but in contrary to Lim et al. (2010) results who recognised threshold linear model as an appropriate model. Model selection was different in these references. Adams et al. (1983) selected the quadratic model based on previous results which were not on sugar beet crop. Sayili and Akca (2004) also selected the appropriate model based on coefficient of determination and standard error and did not test

the residual normality. Further, the models which fitted by them were based on questionnaire results not field results. The results of the study by Lim et al. (2010) were also based on fitted models on values obtained from pot experiment. Although the optimal rate of N in threshold linear model occurred in the point of coincidence between linear production function and the fixed linear part of the curve (Cerrato and Blackmer 1990), the optimum rate of N in 2004 and 2005

were 157.8 and 131.4 kg/ha, respectively, which was a fixed rate (did not depend on the variation in fertilizer cost to crop cost). Therefore, based on governmental and free markets prices in 2012, the optimum fertilizer rate was not different from previous years. In a study by Lim et al. (2010), using threshold linear model, the optimum rate of N fertilizer was 138 kg/ha which did not correspond with the results of the year 2004. Overall, this model showed lower rates compared with the other models in this study.

In 2004, the quadratic threshold model did not give the optimum fertilizer rate but in 2005, the optimum fertilizer rate (158.2 kg/ha) was obtained using this model which was lower than the rate estimated by quadratic polynomial model. Using quadratic polynomial, the economic, optimum rate of N fertilizer in 2004 and 2005 were 235.8 and 248.9 kg/ha, respectively. Using this model, the economic, optimum rate of N fertilizer in 2012 based on year 2004 with approved and free markets rates were 234.7 and 225.1 kg/ha, respectively, and based on the year 2005, 247.9 and 240.8 kg/ha, respectively. Results are comparable with Adams et al. (1983) results who obtained the rate of N application for economic production in three years experiment with 214, 205 and 240 kg/ha, respectively but are in contrary to the rate (321.07 kg/ha) estimated by Sayili and Akca (2004).

Review of the economic optimum fertilizer rates show that with further increase of fertilizer cost, the optimum rate dropped so that based on quadratic polynomial, in 2004 it was decreased to 235.8 kg/ha and in 2012 based on approved price decreased to 234.7 kg/ha, and based on free markets price decreased to 225.1 kg/ha. According to approved price of urea fertilizer between 2004 and 2012, reduction in economic, optimum rate was only 1 kg/ha. However, with considering the free markets price in 2012, this rate will reach to 11 kg/ha. With comparing the years of 2005 and 2012, the difference between economic, optimum rate based on approved rate was 1 kg/ha and with considering the free rate in 2012, it was about 8 kg/ha. The effects of approved and free markets price of urea fertilizer on economic, optimum N application in 2012 showed that in this year, through considering quadratic models of 2004 and 2005, the distance between economic optimum application rate of fertilizer based on approved and free markets price were 9.6 and 7.1 kg/ha, respectively. Based on these results, while increasing fertilizer cost based on approved rates for 2004, 2005 and 2012 only led to 1 kg/ha decrease

in the efficiency of fertilizer application, in year 2012, the difference between approved fertilizer price and free markets price resulted in 7-10 kg/ha decrease in fertilizer rate.

The quadratic polynomial threshold estimated the economic, optimum fertilizer rate of 158.2 kg/ha for the year 2004. In 2012, this rate was 157.9 and 155.0 kg/ha for approved and free markets prices, respectively. With square root model in 2004, the economic, optimum rate of fertilizer was 264.1 kg/ha. In 2012, with this model and governmental price, the economic, optimum rate of fertilizer was 280.7 kg/ha. In this model, with increase of the price, instead of expecting a decrease in fertilizer application, an increase of 16.6 kg/ha was observed. Table 4 shows that in square root model, the economic, optimum rate of fertilizer was higher than the maximum rate which indicated that the square root model was not an appropriate model for estimating the optimum fertilizer rate. Overall, the results show that in threshold linear, quadratic polynomial threshold, quadratic polynomial and square root models, the economic, optimum rate of fertilizer increased which was in agreement with Cerrato and Blackmer (1990), Donald et al. (1994), and Mortensen and Beattie (2005) results. Although in the results of Sumelius et al. (2002), Tageldin and El-Gizawy (2005), and Mortensen and Beattie (2005), the economic, optimum rate derived from threshold linear model was smaller than the quadratic equation threshold and the quadratic threshold but square root and exponential models showed smaller rate than the quadratic equation and the quadratic threshold. In both years of the experiment, quadratic and square root models were appropriate for description of sugar beet production function to N fertilizer based on applied statistical parameters but for estimation of economic, optimum rate of N fertilizer in sugar beet agronomy, quadratic model was selected as appropriate. The results of this study showed that the quadratic polynomial model is appropriate for description of production function and economic, optimum rate in sugar beet production. It also describes that the best model which describes production function based on N rate, is not necessarily an appropriate model for estimating the economic, optimum of applied fertilizer.

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